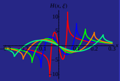
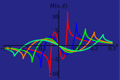


Regge behavior & GPDs at the border point $x = \xi$

A.V. Radyushkin

Physics Department, Old Dominion University
&
Theory Center, Jefferson Lab

Talk at GARYFEST, October 29,2010



Hadrons in Terms of Quarks and Gluons

Situation in hadronic physics:

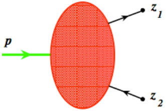
- All relevant particles established
- QCD Lagrangian is known
- Need to understand how QCD works

How to relate hadronic states $|p, s\rangle$

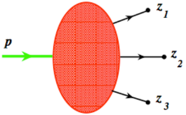
to quark and gluon fields $q(z_1)$, $q(z_2)$, $\ldots$?

Standard way: use matrix elements

$$\langle\, 0\,|\,\bar{q}_\alpha(z_1)\, q_\beta(z_2)\,|\,M(p), s\,\rangle \;\;,\;\; \langle\, 0\,|\,q_\alpha(z_1)\; q_\beta(z_2)\, q_\gamma(z_3)|\,B(p), s\,\rangle$$

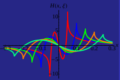


Meson–quark matrix element



Baryon–quark matrix element

- Can be interpreted as hadronic wave functions



Light-cone formalism

- Describe hadron by Fock components in infinite-momentum frame

For nucleon

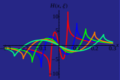
$$|P\rangle = |q(x_1P, k_{1\perp})\, q(x_2P, k_{2\perp})\, q(x_3P, k_{3\perp})\rangle$$
$$+ |qqqG\rangle + |qqq\bar{q}q\rangle + |qqqGG\rangle + \dots$$

- x_i : momentum fractions

$$\sum_i x_i = 1$$

- $k_{i\perp}$: transverse momenta

$$\sum_i k_{i\perp} = 0$$

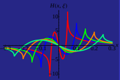


Problems of LC Formalism

- In principle: Solving bound-state equation

$$H|P\rangle = E|P\rangle$$

  one gets $|P\rangle$ which gives complete information about hadron structure
- In practice: Equation (involving infinite number of Fock components) has not been solved and is unlikely to be solved in near future
- Experimentally: LC wave functions are not directly accessible
- Way out: Description of hadron structure in terms of phenomenological functions



Phenomenological Functions

“Old” functions:

- Form Factors
- Usual Parton Densities
- Distribution Amplitudes

“New” functions:

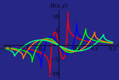
Generalized Parton Distributions (GPDs)

GPDs $=$ Hybrids of

Form Factors, Parton Densities and Distribution Amplitudes

“Old” functions

are limiting cases of “new” functions



Form Factors

Form factors are defined through matrix elements

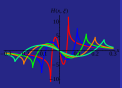
of electromagnetic and weak currents between hadronic states

Nucleon EM form factors:

$$\langle p', s' \,|\, J^\mu(0) \,|\, p, s \rangle = \bar{u}(p', s') \left[\gamma^\mu F_1(t) + \frac{\Delta^\nu \sigma^{\mu\nu}}{2m_N} F_2(t) \right] u(p, s)$$

$(\Delta = p - p', t = \Delta^2)$

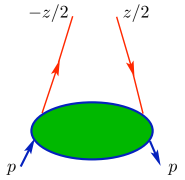
- Electromagnetic current
$$J^\mu(z) = \sum_{f(lavor)} e_f \bar{\psi}_f(z) \gamma^\mu \psi_f(z)$$
- Helicity non-flip form factor
$$F_1(t) = \sum_f e_f F_{1f}(t)$$
- Helicity flip form factor
$$F_2(t) = \sum_f e_f F_{2f}(t)$$



Usual Parton Densities

Parton Densities are defined through forward matrix elements

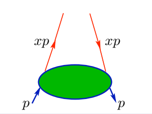
of quark/gluon fields separated by lightlike distances



Unpolarized quarks case:

$$\langle p \,|\, \bar{\psi}_a(-z/2)\gamma^\mu \psi_a(z/2) \,|\, p \rangle \big|_{z^2=0}$$
$$= 2p^\mu \int_0^1 \left[e^{-ix(pz)} f_a(x) - e^{ix(pz)} f_{\bar{a}}(x) \right] dx$$

Momentum space interpretation



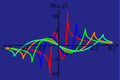
$f_{a(\bar{a})}(x)$ is probability

to find a $(\bar{a})$ quark with momentum xp

Local limit $z=0$

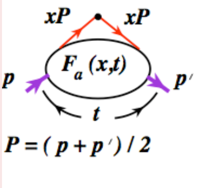
$\Rightarrow$ sum rule

$\int_0^1 [f_a(x) - f_{\bar{a}}(x)]\, dx = N_a$ for valence quark numbers



Nonforward Parton Densities (Zero Skewness GPDs)

Combine form factors with parton densities



$$F_1(t) = \sum_a F_{1a}(t)$$

$$F_{1a}(t) = \int_0^1 \mathcal{F}_{1a}(x,t)\,dx$$

Flavor components of form factors

$$\mathcal{F}_{1a}(x,t) \equiv e_a[\mathcal{F}_a(x,t) - \mathcal{F}_{\bar{a}}(x,t)]$$

Forward limit $t = 0$

$$\mathcal{F}_{a(\bar{a})}(x,t=0) = f_{a(\bar{a})}(x)$$

Interplay between x and t dependences

Simplest factorized ansatz

$$\mathcal{F}_a(x,t) = f_a(x) F_1(t)$$

satisfies both forward and local constraints

Forward constraint

$$\mathcal{F}_a(x, t=0) = f_a(x)$$

Local constraint

$$\int_0^1 [\mathcal{F}_a(x,t) - \mathcal{F}_{\bar{a}}(x,t)] dx = F_{1a}(t)$$

Reality is more complicated:

LC wave function with Gaussian $k_\perp$ dependence

$$\Psi(x_i, k_{i\perp}) \sim \exp\left[-\frac{1}{\lambda^2}\sum_i \frac{k_{i\perp}^2}{x_i}\right]$$

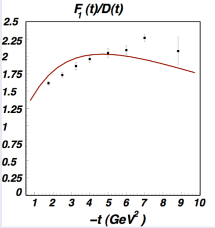
suggests

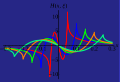
$$\mathcal{F}_a(x,t) = f_a(x) e^{\bar{x}t/2x\lambda^2}$$

$f_a(x)$=experimental densities

Adjusting λ^2 to provide

$$\langle k_\perp^2 \rangle \approx (300\mathrm{MeV})^2$$





Regge-type models for NPDs ($\xi = 0$ GPDs)

"Regge" improvement:

$$f(x) \sim x^{-\alpha(0)}$$
$$\Rightarrow \mathcal{F}(x,t) \sim x^{-\alpha(t)}$$
$$\Rightarrow \mathcal{F}(x,t) = f(x) x^{-\alpha' t}$$

Accomodating quark counting rules:

$$\mathcal{F}(x,t) = f(x) x^{-\alpha' t(1-x)}|_{x \to 1}$$
$$\sim f(x) e^{\alpha'(1-x)^2 t}$$

Does not change small-x behavior but provides

$f(x)|_{x\to 1}$ vs. $F(t)|_{t\to\infty}$ interplay:

$$f(x) \sim (1-x)^n \Rightarrow F_1(t) \sim t^{-(n+1)/2}$$

Note: no pQCD involved in these counting rules!

Extra $1/t$ for $F_2(t)$

can be produced by taking

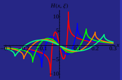
$$\mathcal{E}_a(x,t) \sim (1-x)^2 \mathcal{F}_a(x,t)$$

for "magnetic" NPDs

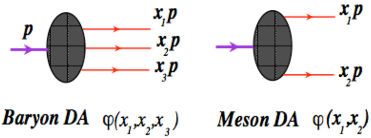
More general:

$\mathcal{E}_a(x,t) \sim (1-x)^{\eta_a} \mathcal{F}_a(x,t)$

Fit : $\eta_u = 1.6$, $\eta_d = 1$



Distribution Amplitudes



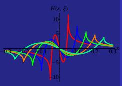
DAs may be interpreted as

- LC wave functions integrated over transverse momentum
- Matrix elements $\langle 0|\mathcal{O}|p\rangle$ of LC operators

For pion (π^+):

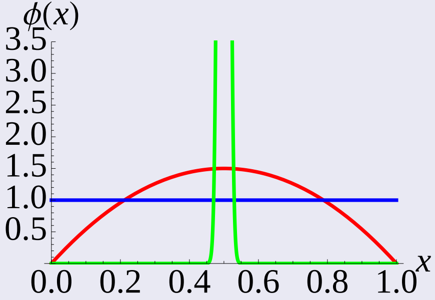
$$\langle\, 0\,|\, \bar{\psi}_d(-z/2)\gamma_5\gamma^\mu\psi_u(z/2)\,|\,\pi^+(p)\,\rangle\big|_{z^2=0}$$
$$= ip^\mu f_\pi \int_{-1}^{1} e^{-i\alpha(pz)/2}\varphi_\pi(\alpha)\, d\alpha$$

with $\alpha = x_1 - x_2$ or $x_1 = (1+\alpha)/2,\; x_2 = (1-\alpha)/2$



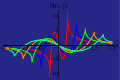
Models for Meson Distribution Amplitudes

Simple power models, $r = 0, 1, 1000$

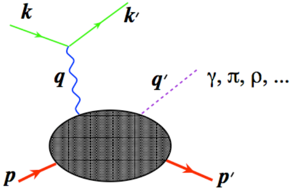


Functional Form:

$\varphi_r(x) \sim [x(1-x)]^r$ or $\phi_r(\alpha) \sim (1-\alpha^2)^r$



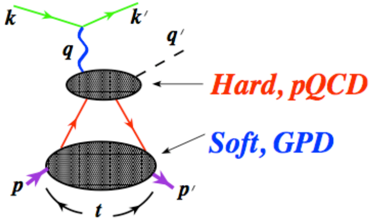
Hard Electroproduction Processes: Path to GPDs



Deeply Virtual Photon and Meson Electroproduction:

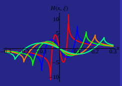
Attempt to use perturbative QCD to extract new information about hadronic structure

pQCD Factorization

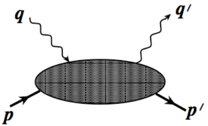


Hard kinematics:

Q^2 is large
$s \equiv (p+q)^2$ is large
$Q^2/2(pq) \equiv x_{\rm Bj}$ is fixed
$t \equiv (p-p')^2$ is small



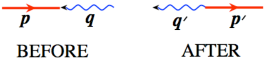
Deeply Virtual Compton Scattering



Kinematics

Total CM energy $s = (q+p)^2 = (q'+p')^2$
LARGE: Above resonance region
Initial photon virtuality $Q^2 = -q^2$
LARGE $(> 1\,\text{GeV}^2)$
Invariant momentum transfer $t = \Delta^2 = (p-p')^2$
SMALL $(\ll 1\text{GeV}^2)$

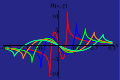
- Picture in $\gamma^* N$ CM frame



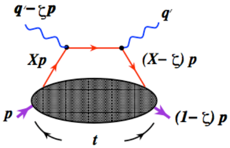
- Virtual photon momentum $q = q' - x_{Bj}p$ has component $-x_{Bj}p$ canceled by momentum transfer Δ
- $\Rightarrow$ Momentum transfer Δ has longitudinal component

$$\Delta^+ = x_{Bj}p^+ \quad , \quad x_{Bj} = \frac{Q^2}{2(pq)}$$

- "Skewed" Kinematics: $\Delta^+ = \zeta p^+$, with $\zeta = x_{Bj}$ for DVCS



Parton Picture for DVCS



Nonforward parton distribution

$\mathcal{F}_\zeta(X;t)$ depends on
X : fraction of p^+
ζ : skeweness
t : momentum transfer

- In forward $\Delta = 0$ limit

$$\mathcal{F}^a_{\zeta=0}(X, t=0) = f_a(X)$$

- Note: $\mathcal{F}^a_{\zeta=0}(X, t=0)$ comes from Exclusive DVCS Amplitude, while $f_a(X)$ comes from Inclusive DIS Cross Section
- Zero skeweness $\zeta = 0$ limit for nonzero t corresponds to nonforward parton densities

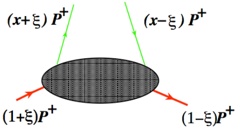
$$\mathcal{F}^a_{\zeta=0}(X, t) = \mathcal{F}^a(X, t)$$

- Local limit: relation to form factors

$$(1-\zeta/2)\int_0^1 \mathcal{F}^a_\zeta(X,t)\,dX = F^a_1(t)$$

Off-forward Parton Distributions

Momentum fractions taken wrt average momentum $P = (p + p')/2$



4 functions of x, ξ, t:

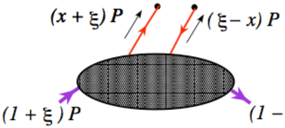
$H, E, \widetilde{H}, \widetilde{E}$

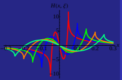
wrt hadron/parton helicity flip

$+/+, -/+, +/-, -/-$

- Skeweness $\xi \equiv \Delta^+/2P^+$ is $\xi = x_{Bj}/(2 - x_{Bj})$
- 3 regions:

$\xi < x < 1$	$\sim$	quark distribution
$-1 < x < -\xi$	$\sim$	antiquark distribution
$-\xi < x < \xi$	$\sim$	distribution amplitude for $N \to \bar{q}qN'$

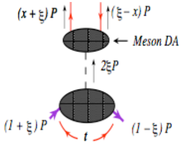




Modeling GPDs

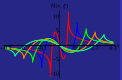
Two approaches are used:

- Direct calculation in specific dynamical models: bag model, chiral soliton model, light-cone formalism , etc.
- Phenomenological construction based on relation of GPDs to usual parton densities $f_a(x), \Delta f_a(x)$ and form factors $F_1(t), F_2(t), G_A(t), G_P(t)$
- Formalism of Double Distributions is often used to get self-consistent phenomenological models



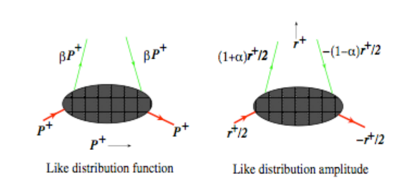
Meson exchange contribution

- GPD $\widetilde{E}(x, \xi; t)$ is related to pseudoscalar form factor $G_P(t)$ and is dominated for small t by pion pole term $1/(t - m_\pi^2)$
- Dependence of $\widetilde{E}(x, \xi; t)$ on x is given by pion distribution amplitude $\varphi_\pi(\alpha)$ taken at $\alpha = x/\xi$

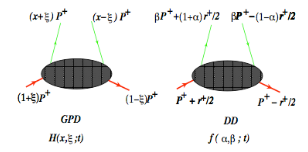


Double Distributions

"Superposition" of P^+ and r^+ momentum fluxes



Connection with OFPDs

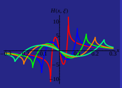


Basic relation between fractions

$$x = \beta + \xi\alpha$$

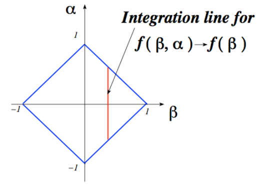
- Forward limit $\xi = 0, t = 0$ gives usual parton densities

$$\int_{-1+|\beta|}^{1-|\beta|} f_a(\beta, \alpha; t = 0)\, d\alpha = f_a(\beta)$$



Getting GPDs from DDs

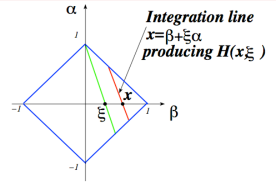
DDs live on rhombus $|\alpha| + |\beta| \le 1$



"Munich" symmetry:

$$f_a(\beta, \alpha; t) = f_a(\beta, -\alpha; t)$$

Converting DDs into GPDs



GPDs $H(x, \xi)$ are obtained from DDs $f(\beta, \alpha)$

by scanning DDs
at ξ-dependent angles
$\Rightarrow$ DD-tomography

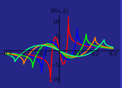


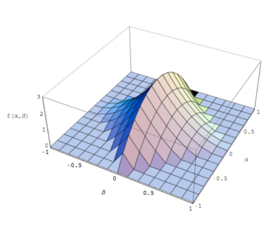
Illustration of DD→GPD conversion

Factorized model for DDs:

($\sim$ usual parton density in β-direction) $\otimes$
($\sim$ distribution amplitude in α-direction)

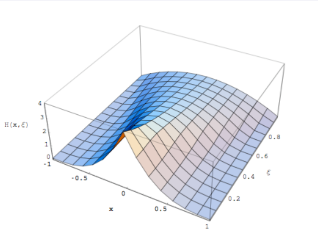
Toy model for double distribution

$$f(\beta,\alpha) = 3[(1-|\beta|)^2 - \alpha^2]\,\theta(|\alpha|+|\beta|\le 1)$$

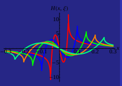


- Corresponds to toy "forward" distribution
$$f(\beta) = (1-|\beta|)^3$$

GPD $H(x,\xi)$ resulting from toy DD



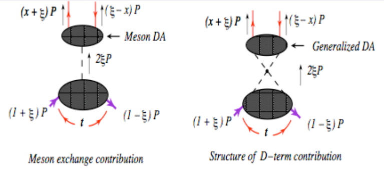
- For $\xi = 0$ reduces to usual parton density
- For $\xi = 1$ has shape like meson distribution amplitude



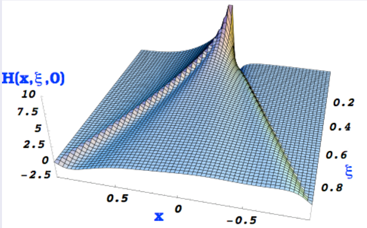
Realistic Model for GPDs based on DDs

- DD modeling misses terms invisible in the forward limit:
  - Meson exchange contributions
  - D-term, which can be interpreted as σ exchange
- Inclusion of D-term induces nontrivial behavior in $|x| < \xi$ region

Meson and D-term terms



DD + D-term model



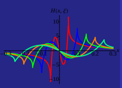
- Profile model for DDs: $f_a(\beta, \alpha) = f_a(\beta) h_a(\beta, \alpha)$

Normalization

$$\int_{-1}^{1} d\alpha \, h(\beta, \alpha) = 1$$

Guarantees forward limit

$$\int_{-1}^{1} d\alpha \, f(\beta, \alpha) = f(\beta)$$

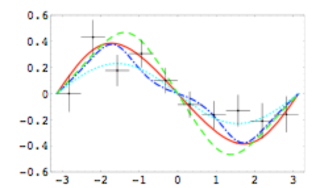


DD Profile

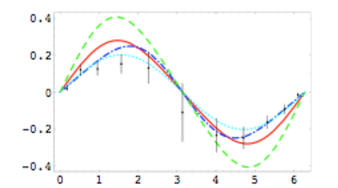
- General form of model profile $h(\beta, \alpha) = \frac{\Gamma(2+2b)}{2^{2b+1}\Gamma^2(1+b)} \frac{[(1-|\beta|)^2-\alpha^2]^b}{(1-|\beta|)^{2b+1}}$
- Power b is parameter of the model
- $b = \infty$ gives $h(\beta, \alpha) = \delta(\alpha)$ and $H(x, \xi) = f(x)$
- Single-Spin Asymmetry

$$A_{LU}(\varphi) = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

HERMES Data

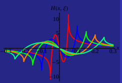


JLab CLAS Data



- Models:

Red:	$b_{\text{val}} = 1$	$b_{\text{sea}} = \infty$
Green:	$b_{\text{val}} = 1$	$b_{\text{sea}} = 1$
Blue:	$b_{\text{val}} = \infty$	$b_{\text{sea}} = \infty$



Models with Regge behavior of $f(\beta)$

- Szczepaniak et al: constructed model equivalent to

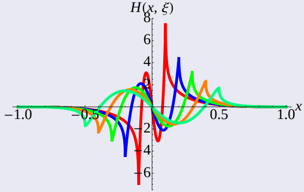
$$H(x,\xi) = x \int_{\Omega} d\beta \, \frac{f(\beta)}{\beta(1-|\beta|)} \, \delta(x - \beta - \xi\alpha)$$

- Corresponds to $b=0$ flat profile $h(\beta,\alpha) = \frac{1}{2(1-|\beta|)}$
- Regge ansatz $f(\beta) \sim |\beta|^{-a}$ gives singularity at border point $x = \xi$

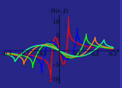
$$H(x,\xi)|_{x\sim\xi} \sim \left| \frac{x-\xi}{1-\xi} \right|^{-a} \quad \text{Bad}: \; A_{\text{DVCS}} \sim \int_{-1}^{1} \frac{dx}{x - \xi + i\epsilon} H(x,\xi)$$

- Flat profile follows from hard $1/k_i^2$ behavior of parton-hadron amplitude $T(p_1, p_2; k_1, k_2)$
- Changing to faster $(1/k_i^2)^{b+1}$ fall-off gives b-profile
- No singularities with $b \geq a$

b=1 DD with $a = 0.5$ Regge PDFs



$\xi = 0.1, 0.2, 0.3, 0.4, 0.5$



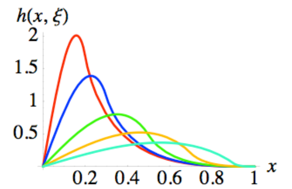
Early model with Regge behavior of $f(\beta)$

- Direct model $H(x,\xi) = \int_\Omega d\beta\, f(\beta)\, h_b(\beta,\alpha)\, \delta(x-\beta-\xi\alpha)$ with $b=1$

$$
\begin{aligned}
H(x,\xi)|_{|x|\geq\xi} &= \frac{1}{\xi^3}\left(1-\frac{a}{4}\right)\left\{\left[(2-a)\xi(1-x)(x_+^{2-a}+x_-^{2-a})\right.\right.\\
&+ \left.\left.(\xi^2-x)(x_+^{2-a}-x_-^{2-a})\right]\ \theta(x)-(x\rightarrow -x)\right\}\\
H(x,\xi)|_{|x|\leq\xi} &= \frac{1}{\xi^3}\left(1-\frac{a}{4}\right)\left\{x_+^{2-a}[(2-a)\xi(1-x)+(\xi^2-x)]\right.\\
&- \left.(x\rightarrow -x)\right\}
\end{aligned}
$$

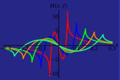
- $f(\beta) \sim \beta^{-a}(1-\beta)^3$
- $x_+ = (x+\xi)/(1+\xi)$
- $x_- = (x-\xi)/(1-\xi)$
- $\sim |x-\xi|^{2-a}$+const behavior for $x \sim \xi$

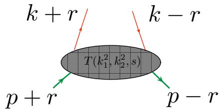
b=1 DD with Regge PDFs



$\xi = 0.2, 0.3, 0.5, 0.7, 0.9$

Basics of the "Regge-blob" model

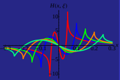




- Quark-hadron scattering amplitude is modeled by

$$\gamma_\mu k^\mu \frac{1}{(m_1^2-(k+r)^2)^{n_1+1}} \frac{1}{(m_2^2-(k-r)^2)^{n_2+1}} T((p-k)^2)$$

- Dirac structure $\gamma_\mu k^\mu$ is necessary to provide EM gauge invariance of DVCS amplitude
- Modified propagators soften quark-hadron vertices

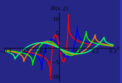


Combining with the dispersion relation

- Model is based on

$$H(x,\xi)\,P^+ \sim \int k^+ \frac{\delta(x - k^+/P^+)\,d^4k}{[m_1^2 - (k+r)^2]^{N_1+1}[m_2^2 - (k-r)^2]^{N_2+1}}$$
$$\times \int_0^\infty d\sigma \rho(\sigma) \left\{ \frac{1}{\sigma - (P-k)^2} - \frac{1}{\sigma} \right\}$$

- First line: modified propagators providing softer quark-hadron vertices (eventually $N_1 = N_2 \equiv N$) can be obtained by $(d/dm_i^2)^{N_i}$
- Second line: quark-hadron scattering amplitude in (subtracted) dispersion relation representation
- Choosing $\rho(\sigma)$ to get Regge $\sim s^\alpha$ behavior in $s = (P-k)^2$



How profile factor appears

- In Feynman parameters:

$$H(x,\xi) \sim \int_0^\infty d\sigma\, \rho(\sigma) \int_0^1 \frac{(x_3 P^+ + (x_2 - x_1) r^+)/P^+}{(x_3\sigma + x_1 m_1^2 + x_2 m_2^2)^{n_1+n_2+1}}\, x_1^{n_1}\, x_2^{n_2}\, [dx] \left\{ \delta\left(x - x_3 - (x_2 - x_1)\xi\right) - \frac{\delta\left(x - (x_2 - x_1)\xi\right)}{(x_1 + x_2)^2} \right\}$$

- $[dx] = dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3)$
- In DD representation we should have $\beta P^+ + \alpha r^+$, which gives

$$x_1 = (1 - \beta - \alpha)/2 \;\;,\;\; x_2 = (1 - \beta + \alpha)/2$$

- For equal $N_i = N$: profile factor

$$(x_1 x_2)^N = [(1-\beta)^2 - \alpha^2]^N / 2^{2N}$$

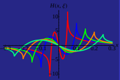
- Note: taking $m_1 = m_2 = m$ before differentation gives $(x_1 + x_2)^{2N}$ after it, i.e. $(1-\beta)^{2N}$ $\Rightarrow$ flat profile in α direction!

Criticism of “Indiana model”

- Little bit of algebra:

$$\left(\frac{d}{dm^2}\right)^2 \frac{1}{(m^2-k_1^2)(m^2-k_2^2)} = \frac{1}{(m^2-k_1^2)^3(m^2-k_2^2)} + \frac{2}{(m^2-k_1^2)^2(m^2-k_2^2)^2} + \frac{1}{(m^2-k_1^2)(m^2-k_2^2)^3}$$

- Note: two terms have unmodified propagators $\Rightarrow$ no softening of one of the quark-hadron vertices
- But quark-nucleon vertex cannot be pointlike!
- More formal objection: factorization proofs through operator product expansion imply QCD equation of motion $\gamma_\mu D^\mu \psi_q = 0$, while pointlike qN vertex corresponds to $\gamma_\mu D^\mu \psi_q = \Psi_N$
- Stick to model with both quark propagators modified



Softened model

- In GPD variables $\beta P^+ + \alpha r^+ = x P^+$, so

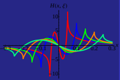
$$H(x,\xi) \sim \frac{x}{2^{2n+1}} \int_0^\infty d\sigma\, \rho(\sigma) \int_0^1 d\beta \int_{-1+\beta}^{1-\beta} d\alpha\, \frac{[(1-\beta)^2 - \alpha^2]^n}{(\beta\sigma + (1-\beta)m^2)^{2n+1}} \left\{ \delta\left(x - \beta - \alpha\xi\right) - \frac{\delta\left(x - \alpha\xi\right)}{(1-\beta)^2} \right\}$$

- Usual (forward) parton distribution corresponds to $\xi = 0$

$$H(x,\xi=0) = \frac{x}{2^{2n+1}} \int_0^\infty d\sigma\, \rho(\sigma) \int_0^1 d\beta \int_{-1+\beta}^{1-\beta} \frac{[(1-\beta)^2 - \alpha^2]^n\, d\alpha}{(\beta\sigma + (1-\beta)m^2)^{2n+1}} \times \left\{ \delta\left(x - \beta\right) - \frac{\delta\left(x\right)}{(1-\beta)^2} \right\}$$

- Note: $x\delta(x) = 0$, thus

$$f(x) = \frac{(n!)^2}{(2n+1)!}\, x\,(1-x)^{(2n+1)} \int_0^\infty \frac{d\sigma\, \rho(\sigma)}{(x\sigma + (1-x)m^2)^{2n+1}}$$



Softened model, contd.

- Substituting σ-integral by forward distribution gives for GPD

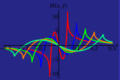
$$H(x,\xi) = \frac{x}{2^{2n+1}} \frac{(2n+1)!}{(n!)^2} \int_0^1 d\beta \int_{-1+\beta}^{1-\beta} d\alpha \frac{[(1-\beta)^2-\alpha^2]^n}{(1-\beta)^{2n+1}} \frac{f(\beta)}{\beta} \times \left\{ \delta\left(x-\beta-\alpha\xi\right) - \frac{\delta\left(x-\alpha\xi\right)}{(1-\beta)^2} \right\}$$

- Normalized profile function:

$$h_n(\beta,\alpha) \equiv \frac{1}{2^{2n+1}} \frac{(2n+1)!}{(n!)^2} \frac{[(1-\beta)^2-\alpha^2]^n}{(1-\beta)^{2n+1}}$$

- Result:

$$\frac{H(x,\xi)}{x} = \int_0^1 d\beta \int_{-1+\beta}^{1-\beta} d\alpha \frac{f(\beta)}{\beta} h_n(\beta,\alpha) \times \left\{ \delta\left(x-\beta-\alpha\xi\right) - \frac{\delta\left(x-\alpha\xi\right)}{(1-\beta)^2} \right\}$$



New version of DD anasatz

- Regularized DD ansatz:

$$\frac{H(x,\xi)}{x} = \int_0^1 d\beta \int_{-1+\beta}^{1-\beta} d\alpha\, \delta\left(x-\beta-\alpha\xi\right) \times \left\{ f(\beta,\alpha) - \delta(\beta)\int_0^{1-|\alpha|} d\gamma\, \frac{f(\gamma,\alpha)}{(1-\gamma)^2} \right\}$$

  with

$$f(\beta,\alpha) = f(\beta)\, h_n(\beta,\alpha)/\beta$$

- This representation includes D-term

$$D(\alpha) = \alpha \int_0^{1-|\alpha|} d\beta\, \frac{f(\beta)}{\beta}\, h(\beta,\alpha) \left\{ 1 - \frac{1}{(1-\beta)^2} \right\}$$

- Total double distribution

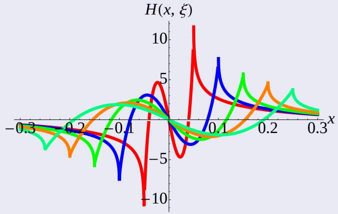
$$F(\beta,\alpha) = [f(\beta,\alpha]_+ + \delta(\beta) D(\alpha)$$

- Usual “plus” prescription

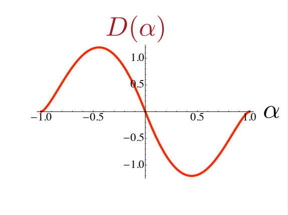
$$[f(\beta,\alpha)]_+ \equiv f(\beta,\alpha) - \delta(\beta)\int_0^{1-|\alpha|} d\gamma\, f(\gamma,\alpha)$$

Results for $n = 1$ profile $\sim [(1-\beta)^2 - \alpha^2]$

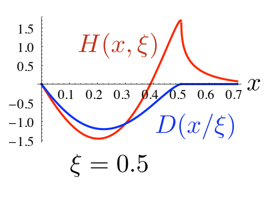
$H(x,\xi)$; $\xi = 0.05, 0.1, 0.15, 0.2, 0.25$



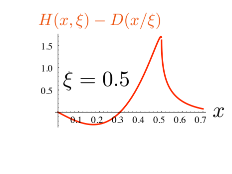
D-term



Comparison of GPD and D-term

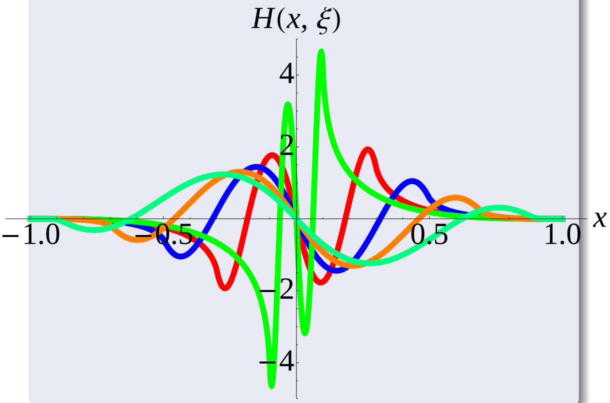


Difference of GPD and D-term



Results for $n = 2$ profile $\sim [(1-\beta)^2 - \alpha^2]^2$

$H(x,\xi); \xi = 0.1, 0.3, 0.5, 0.7, 0.9$



- First derivative $dH(x,\xi)/dx$ is continuous at $x = \xi$

Conclusion

Happy Birthday Gary!